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Fast reconnection of weak magnetic fields

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Fast magnetic reconnection refers to annihilation or topological rearrangement of magnetic fields on a timescale that is independent (or nearly independent) of the plasma resistivity. The resistivity of astrophysical plasmas is so low that reconnection is of little practical interest unless it is fast. Yet, the theory of fast magnetic reconnection is on uncertain ground, as models must avoid the tendency of magnetic fields to pile up at the reconnection layer, slowing down the flow. In this paper it is shown that these problems can be avoided to some extent if the flow is three dimensional. On the other hand, it is shown that in the limited but important case of incompressible stagnation point flows, every flow will amplify most magnetic fields. Although examples of fast magnetic reconnection abound, a weak, disordered magnetic field embedded in stagnation point flow will in general be amplified, and should eventually modify the flow. These results support recent arguments against the operation of turbulent resistivity in highly conducting fluids. © *1998 American Institute of Physics.* [S1070-664X(98)04801-0]

I. INTRODUCTION

Magnetic fields in astrophysical systems are almost completely frozen to the plasma, because the magnetic Reynolds number R_m, the ratio of the Ohmic decay time to the dynamical time, is extremely large; of order 10^{15} to 10^{21} for interstellar fields and 10^8 to 10^{10} for stellar fields, for example. Any departure from frozen in behavior—i.e. any reconnection of the magnetic fieldlines—is a finite conductivity effect. If magnetic reconnection occurs on a timescale which is independent of R_m, the reconnection is said to be fast. Petschek's model[1] at its maximum rate is almost fast, depending only logarithmically on R_m. The Sweet and Parker time independent model[2,3] is slow, with the reconnection rate scaling as $R_m^{-1/2}$. Linear tearing modes[4] grow at a rate scaled by $R_m^{-3/5}$ and are also slow.

Given the large value of R_m in many astrophysical situations, reconnection can be of little practical importance unless it is fast. Therefore, the problem of developing models for fast magnetic reconnection is of considerable interest.

Clark[5] and later Moffatt[6] developed an analytical model for almost fast magnetic reconnection based on two dimensional (2D), incompressible flow near a hyperbolic stagnation point. I refer to this model as 2DHS throughout the text. The magnetic field is initially amplified by induction, and then decays resistively at a superexponential rate. Strauss[7] showed that this simple model describes a numerical simulation of magnetic reconnection in a current sheet.

The 2DHS model is kinematic, in the sense that the flow field is prescribed. During the transient amplification phase, magnetic forces can become very large, scaling as $R_m^{3/2}$, and one concludes that the reconnection is quenched or greatly modified by these large forces, even though they act for only a short time. The dynamical, steady state reconnection model of Craig and Henton[8] shows magnetic flux pileup and large pressure gradients near the stagnation point of the flow—which is also an X-point of the magnetic field—and might represent the outcome of the 2DHS model if magnetic forces were accounted for. See References 5, 9 and 10 for other discussions of the role of large pressure gradients in sustaining the flow in fieldline reconnection.

The 2DHS model exemplifies a generic difficulty of reconnection theory: in order that resistive effects occur rapidly, the magnetic field must develop structure on small spatial scales which are proportional to a positive power of the plasma resistivity η. These small spatial scales generally imply large Lorentz forces, scaling as a negative power of η. But, fast reconnection requires that the plasma velocity remain independent of η.

The difficulties associated with the 2DHS model can be made to disappear in three dimensional flows. More generally, it turns out to be relatively easy to characterize the action of *any* linear stagnation point flow on *any* magnetic field, and to set forth conditions under which the fields decay and Lorentz forces do not grow, so that if the kinematic reconnection theory is self-consistent initially then it remains so. This allows one to estimate, for example, the degree of reconnection of a weak magnetic field in a turbulent fluid. It may also be a useful preliminary step toward developing fully dynamical models of reconnection, which apply when the fields are strong.

In Sec. II of this paper I review the 2DHS model and generalize it to three dimensions. In Sec. III I develop the solution for arbitrary fields in arbitrary hyperbolic stagnation point flows. In Sec. IV D I summarize the implications of this work for astrophysical reconnection, and for the operation of turbulent resistivity, which is often invoked in astrophysics.

II. THE 2D HYPERBOLIC STAGNATION POINT MODEL AND EXTENSION TO THREE DIMENSIONS

Consider a magnetic field $\mathbf{B}=(B(y,t),0,0)$ embedded in a bulk flow $\mathbf{u}$ with a hyperbolic stagnation point at the origin,

$$\mathbf{u}=u'(x,-y,0). \tag{1}$$

Equation (1) should of course be regarded as a local representation of a flow which is globally bounded. Assume the medium has resistivity η and Ohmic diffusivity $\lambda \equiv \eta c^2/4\pi$. The magnetic induction equation,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \lambda \nabla^2 \mathbf{B}, \tag{2}$$

reduces in this case to

$$\frac{\partial B}{\partial t} = u' y \frac{\partial B}{\partial y} + u' B + \lambda \frac{\partial^2 B}{\partial y^2}. \tag{3}$$

Taking as the initial condition

$$B(y,0) = B_0 \sin k_0 y, \tag{4}$$

Eq. (3) has the solution

$$B(y,t) = B_0 e^{u't - R_m^{-1}(e^{2u't}-1)} \sin k_0 e^{u't} y, \tag{5}$$

where the magnetic Reynolds number $R_m \equiv (\lambda k_0^2/2u')^{-1}$ is roughly the ratio of the initial Ohmic decay time to the flow time within one magnetic spatial period. (Although in steady state problems it is standard procedure to define the dynamical time from the advective term in the induction equation, so that R_m is linear in the magnetic lengthscale, the definition used here is natural in the present problem.) It is the dependence on λ, or η, which is most important here. Equation (5) describes a field which is initially amplified by compression as it is swept toward the x axis, while the scale over which the field reverses shrinks exponentially. Eventually Ohmic processes dominate compression and the field begins a phase of superexponential decay. The fieldstrength reaches its maximum, B_{max}, at time t_{max},

$$t_{max} = \frac{1}{2u'} \ln\left(\frac{1}{2} R_m\right);$$

$$B_{max} \approx B_0 \left(\frac{R_m}{2}\right)^{1/2} \sin k_0 \left(\frac{R_m}{2}\right)^{1/2} y, \tag{6}$$

where the approximation consists of assuming $R_m \gg 1$. Thus, the field is amplified by a factor of order $R_m^{1/2}$ or $\eta^{-1/2}$, and the magnetic lengthscale decreases by the same factor. The decay phase is so fast that the field falls to $1/e$ of its initial value in a time of order $(2u')^{-1} \ln(R_m \ln R_m^{1/2})$. In view of the weak dependence of the decay time on R_m, this system is a model of "almost fast" magnetic reconnection. However, it is easy to show that the current density and Lorentz force also peak before decaying, and that the time integrated Lorentz force F_L is of order

$$\int dt F_L \sim \frac{k_0 B_0^2 R_m^{3/2}}{4\pi u'}. \tag{7}$$

The fluid will be slowed down if the time integrated Lorentz force is greater than or equal to the initial momentum density in the fluid, measured, say, at a distance k_0^{-1} from the origin. Therefore, the fluid is decelerated unless $B_0 < \sqrt{(4\pi\rho)}(u'/k_0) R_m^{-3/4}$, which means that the initial Alfvèn Mach number of the flow must exceed $R_m^{3/4}$. Thus, this reconnection model is self-consistent only for extremely weak magnetic fields. It is difficult to imagine applying it in unmodified form to the interstellar medium, where B is close to equipartition with the turbulent gas velocity. Similar objections would arise if the model were applied to the convection zones of stars; even more so in stellar coronae, which are magnetically dominated.

These difficulties can be avoided in a three-dimensional flow. Consider the velocity field,

$$\mathbf{u} = u'(-x, -y, 2z). \tag{8}$$

Fluid is swept in along the x and y axes and ejected along the z axis, and the flow is incompressible (see Reference 11 for a discussion of dynamically consistent flows of this type). Again, assume the magnetic field points in the $\hat{x}$ direction and depends only on y and t. The induction equation (2) is

$$\frac{\partial B}{\partial t} = u' y \frac{\partial B}{\partial y} - u' B + \lambda \frac{\partial^2 B}{\partial y^2}. \tag{9}$$

The solution which satisfies the initial condition (4) is

$$B = B_0 e^{-u't - R_m^{-1}(e^{2u't}-1)} \sin k_0 e^{u't} y. \tag{10}$$

Equation (10) differs from Eq. (5) in that the magnetic field is not compressed in the initial nonresistive phase; instead, the field is weakened by ejection in the z direction even as its spatial scale decreases. Once resistivity becomes important, which happens at about the time t_{max} given in Eq. (6), the spatial scale has decreased to $R_m^{-1/2}$ of its initial value, and decay is superexponential. The current density is nearly flat with time in the nonresistive phase, and the Lorentz force decreases monotonically, so if F_L is initially negligible it remains so.

The results of this section suggest that one could systematically characterize magnetic reconnection near stagnation points in three dimensional flows, and select the flows and fields for which reconnection is fast and unaccompanied by large increases in the Lorentz force. This is taken up in Sec. III.

III. GENERAL STAGNATION POINT FLOW

The resistive induction equation cannot be solved analytically for general flows, even when the velocity is a linear function of the coordinates. The solution to the nonresistive induction equation can, however, be written down exactly, and it is then easy to see whether the magnetic field develops progressively smaller scale structure over time, and how the Lorentz force evolves during the ideal phase.

Much of this section depends on results from linear algebra, and are given in any one of a number of books. The author used the text by Curtis,[12] which includes material on differential equations.

It is convenient to adopt a Lagrangian description of the flow. Label points in the fluid by their initial positions $\mathbf{x_0}$, and write their positions at subsequent times $t>0$ as $\mathbf{x}(\mathbf{x_0},t)$. One can then form the deformation matrix $\mathbf{D}$ at every point and time, which has elements

$$D_{ij} = \frac{\partial x_i}{\partial x_{0j}}. \tag{11}$$

The volume element d^3x changes with time according to $d^3x=|D|d^3x_0$, where $|D|$ is the determinant of $\mathbf{D}$. The induction equation (2) with $\lambda=0$ can then be integrated to give $\mathbf{B}(\mathbf{x},t)$ in terms of the initial field $\mathbf{B}_0$,

$$\mathbf{B}(\mathbf{x},t)=\frac{\mathbf{D}\cdot\mathbf{B}_0(\mathbf{x}_0(\mathbf{x},t))}{|D|}. \tag{12}$$

Equation (12) shows that growth of the field is associated with an increase of the deformation matrix with time, which for incompressible or nearly incompressible flows is associated with shearing or stretching of volume elements. The change in the magnetic lengthscale is also brought about by shear, as reflected in the map of $\mathbf{x}$ back to $\mathbf{x}_0$. In general, this map (or its inverse, the trajectory of a fluid particle) is difficult to construct, but it becomes simple near a stagnation point, taken to be the origin of the coordinate system, at which the Lagrangian equations of motion are

$$\frac{d\mathbf{x}}{dt}=\mathbf{U}\cdot\mathbf{x}, \tag{13}$$

where $\mathbf{U}$ is a 3×3 matrix with constant, real coefficients. The solution of Eq. (13) with the initial condition $\mathbf{x}(0)=\mathbf{x}_0$ is

$$\mathbf{x}(t)=e^{t\mathbf{U}}\cdot\mathbf{x}_0, \tag{14}$$

where the matrix $e^{t\mathbf{U}}$ is defined by its Taylor series,

$$e^{t\mathbf{U}}\equiv\lim_{n\to\infty}\sum_{j=0}^{n}\frac{(t\mathbf{U})^j}{j!}. \tag{15}$$

It is clear from Eqs. (11) and (14) that

$$\mathbf{D}=e^{t\mathbf{U}}, \tag{16}$$

and that the inverse of Eq. (14) is

$$\mathbf{x}_0(\mathbf{x},t)=e^{-t\mathbf{U}}\cdot\mathbf{x}. \tag{17}$$

Assume for the remainder of the paper that $|e^{t\mathbf{U}}|=1$ (incompressible flow). Using this together with Eqs. (14) and (17), Eq. (12) becomes

$$\mathbf{B}(\mathbf{x},t)=e^{t\mathbf{U}}\cdot\mathbf{B}_0(e^{-t\mathbf{U}}\cdot\mathbf{x}). \tag{18}$$

The matrix $e^{t\mathbf{U}}$ is generally not easily computed as it stands. However, by a theorem of linear algebra, the matrix $\mathbf{U}$ is similar to the sum of a diagonal matrix $\mathbf{M}$ and a nilpotent matrix $\mathbf{N}$ which commutes with $\mathbf{M}$,

$$\mathbf{U}=\mathbf{S}(\mathbf{M}+\mathbf{N})\mathbf{S}^{-1}, \tag{19}$$

for some nonsingular matrix $\mathbf{S}$. Because $\mathbf{M}$ and $\mathbf{N}$ commute, this carries over to exponentiation,

$$e^{t\mathbf{U}}=\mathbf{S}e^{t(\mathbf{M}+\mathbf{N})}\mathbf{S}^{-1}. \tag{20}$$

The elements of the diagonal matrix $\mathbf{M}$ are just the roots α_i of the minimal polynomial of $\mathbf{U}$ (the lowest degree polynomial equation satisfied by $\mathbf{U}$), with each root occurring according to its multiplicity. It is readily shown that $e^{t\mathbf{M}}$ is diagonal, with element $(e^{t\mathbf{M}})_{ii}=e^{tM_{ii}}$. Computation of $e^{t\mathbf{N}}$ is also straightforward; since $\mathbf{N}$ is nilpotent, the Taylor series given in Eq. (15) terminates at or before the $\mathbf{N}^3$ term.

Returning to Eq. (18), written now in the form

$$\mathbf{B}(\mathbf{x},t)=\mathbf{S}e^{t(\mathbf{M}+\mathbf{N})}\mathbf{S}^{-1}\cdot\mathbf{B}(\mathbf{S}e^{-t(\mathbf{M}+\mathbf{N})}\mathbf{S}^{-1}\cdot\mathbf{x}), \tag{21}$$

we can make the following observations. The matrix $\mathbf{S}$ is independent of time and does not directly cause growth or decay of the magnetic field or current. The factors $e^{\pm t\mathbf{N}}$ lead at most to algebraic growth or decay of the field and current, by virtue of the finite Taylor series which define these matrices. Exponential growth or decay of the field and its lengthscale comes about only through the action of the $e^{\pm t\mathbf{M}}$ matrices, and the rates of growth/decay are easy to read off; they correspond to the real parts of the roots α_i of the minimal polynomial of the original velocity matrix $\mathbf{U}$, which are also eigenvalues ν_i of $\mathbf{U}$.

There is a constraint on the α_i which arises from the incompressibility of the flow. It follows from Eq. (20) that $|e^{t\mathbf{M}}e^{t\mathbf{N}}|=|e^{t\mathbf{M}}||e^{t\mathbf{N}}|=1$. On the other hand,

$$|e^{t\mathbf{M}}|=\exp\left(t\sum_i n_i\alpha_i\right), \tag{22}$$

where n_i is the multiplicity of the ith root. Since $|e^{t\mathbf{N}}|$ is an algebraic function of t, the real part of the sum in Eq. (22) must be zero.

In order for the lengthscale of the field to shrink to the resistive scale, at least one α_i must have a negative real part. The field grows because of the action of the α_i with positive real part. Because of the remark following Eq. (22), if there is shrinking there must also be stretching, so magnetic fields with arbitrary direction and coordinate dependence are amplified even as their lengthscale shrinks.

The velocity field given by Eq. (8) illustrates these conclusions. In this case, the $\mathbf{U}$ matrix is diagonal. The magnetic field derivatives with respect to x and y grow as $e^{u't}$. The $\hat{x}$ and $\hat{y}$ components of $\mathbf{B}$ shrink by the same factor. The $\hat{z}$ component of $\mathbf{B}$ grows, however, as $e^{2u't}$. The Lorentz force is bounded with time only for fields in the $x-y$ plane, so only fields confined to the $x-y$ plane can undergo fast, kinematic reconnection.

A simple modification of the flow (8) leads to algebraic growth of $\mathbf{B}$. Consider the velocity field

$$\mathbf{u}=u'(-x-y,-y,2z). \tag{23}$$

In this case, direct integration of the equations of motion (13) yields

$$x=x_0e^{-u't}-y_0u'te^{-u't};\quad y=y_0e^{-u't};\quad z=z_0e^{2u't}, \tag{24}$$

while the matrix $\mathbf{U}$ is the direct sum of a diagonal matrix with elements $(-1,-1,2)$ and the nilpotent matrix $\mathbf{N}$ with single nonzero element $N_{12}=-1$, which satisfies the equation $\mathbf{N}^2=0$. In this example, the $\mathbf{U}$ matrix has a doubly degenerate eigenvalue -1, and this corresponds to forcing of x by y at its natural decay rate. This is the origin of the combined exponential and algebraic deformation seen in Eq. (24).

As an example of the action of this flow on a magnetic field, let

$$\mathbf{B}_0(\mathbf{x}_0)=f(x_0)(0,1,0). \tag{25}$$

Using Eq. (12), the field at time t is seen to be

$$\mathbf{B}(\mathbf{x},t)=e^{-u't}f((x+u'yt)e^{u't})(-u't,1,0). \tag{26}$$

Equation (26) shows that the magnetic lengthscale shrinks exponentially and the field decays at the same rate. The fieldlines are also rotated from the $\hat{y}$ to the $\hat{x}$ direction over time, because of the shear in the flow.

Given any incompressible stagnation point flow of the type considered here, is there always an initial magnetic field which is not amplified by it? This question is easily answered in the nondegenerate case, in which all the eigenvalues of **U** are distinct. In this case, the fluid trajectories [which are solutions of Eq. (13)] can be written in the form

$$x_i(t)=a_{ij}(\mathbf{x}_0)e^{\mu_j t}, \tag{27}$$

where the μ_j are eigenvalues of **U**, and the a_{ij} are linear functions of $\mathbf{x}_0$. It is clear from eqs. (11) and (27) that the elements of the deformation matrix are

$$D_{ik}=\frac{\partial a_{ij}}{\partial x_{0k}}e^{\mu_j t}, \tag{28}$$

where the partial derivatives are constants. Therefore, **D** can be written as a sum of constant matrices $\mathbf{D}^{(i)}$, each one multiplied by an exponential function of time,

$$\mathbf{D}=\sum_j \mathbf{D}^{(j)}e^{\mu_j t}. \tag{29}$$

Suppose some μ_i has a positive real part; the condition that an initial field $\mathbf{B}_0$ not grow with $e^{\mu_i t}$ is

$$\mathbf{D}^{(i)}\cdot\mathbf{B}_0=0. \tag{30}$$

That is, $\mathbf{D}^{(i)}$ must be singular, and $\mathbf{B}_0$ must be in the null space of $\mathbf{D}^{(i)}$.

In fact, the $\mathbf{D}^{(j)}$ are all singular. This follows from the condition $|D|=1$. Therefore, in a flow in which only one eigenvalue has a positive real part, it is always possible to find an initial field which is not amplified by the flow. If two eigenvalues μ_i,μ_j have a positive real part, they will not necessarily have overlapping null spaces, and so it may turn out that all fields are amplified by the flow.

Finally, it is worth estimating the amplification of the field and Lorentz force during the ideal phase, bearing in mind that the estimate is based on rough arguments and that there are many special cases. Let the eigenvalue with the largest positive real part be μ_{max}, and let $Re(\mu_{max})$ be the real part of μ_{max}. Let μ_{min} be the eigenvalue with the most negative real part, and $Re(\mu_{min})$ be the real part itself. The rate of reduction in lengthscale l for the field is dominated by μ_{min}, so $l\approx l_0\exp(Re(\mu_{min})t)$. The ideal phase ends at the time t_i when the diffusive timescale becomes equal to the flow timescale,

$$\lambda l^{-2}(t_i)\approx t_{flow}^{-1};\quad t_i\approx\frac{1}{2Re(\mu_{min})}\ln(\lambda l_0^{-2}t_{flow}). \tag{31}$$

The magnetic field is amplified at the rate $\exp(Re(\mu_{max}t))$. According to Eq. (31), the field amplitude at the end of the ideal phase is

$$B\approx B_0(\lambda l_0^{-2}t_{flow})^{Re(\mu_{max})/2Re(\mu_{min})}. \tag{32}$$

In the 2DHS flow given by Eq. (1), $\mu_{max}=1$, $\mu_{min}=-1$, so the amplification factor is proportional to $\lambda^{-1/2}$. In fact, since the real parts of the μ must sum to zero, the exponent in eq. (32) must lie between $-1/4$ and -1, so the field must grow by at least a factor of $\lambda^{-1/4}$ ($\eta^{-1/4}$) during the ideal phase. Examples of similar scalings for a variety of dynamically self-consistent 3D reconnection models are given in Reference 13.

Similar arguments can be made for the growth of the Lorentz force F_L, which scales as B^2/l. Thus, at the end of the ideal phase,

$$F_L\approx F_{L0}(\lambda l_0^{-2}t_{flow})^{-1/2+Re(\mu_{max})/Re(\mu_{min})}. \tag{33}$$

The amplification rate of the Lorentz force lies between λ^{-1} and $\lambda^{-5/2}$.

IV. SUMMARY AND DISCUSSION

In this paper we address the problem of time dependent, fast magnetic reconnection in highly conducting fluids in the kinematic regime, in which the fields are assumed to be so weak that Lorentz forces can be ignored. Reconnection is assumed to take place at a stagnation point, and to be essentially a two stage process. In the first step resistivity can be ignored, and advection of the field by the flow produces structure in the field on very small scales. In the second stage, magnetic gradients are large enough that resistivity is important, and the field decays. The issue is whether the buildup of large Lorentz forces (scaling as an inverse power of the resistivity), which would vitiate the kinematic assumption, can be avoided during the first stage. There are two main results.

First, it is not difficult to find examples of flows and fields in which reconnection takes place at a rate almost independent of the resistivity η, and for which, if the kinematic approximation is self-consistent initially, it remains so. The example discussed in Sec. II is an extension of the 2DHS model in which Lorentz forces scale as $\eta^{-3/2}$. The crucial ingredient is a third component of flow, without which the situation would be hopeless. It is argued in Sec. III that a broad class of incompressible stagnation point flows allow fast magnetic reconnection. These results are encouraging for some astrophysical problems in which the field and flow geometry can be controlled, and the fields are weak, and also provide some insight into the nature of almost fast magnetic reconnection.

The second result is that if the initial magnetic field orientation and coordinate dependence are arbitrary, the field will generally be amplified by the flow. It is the fields which are not amplified which must satisfy special conditions, such as $B_{0z}\equiv 0$ for the flow given in Eq. (8). Since the lengthscales for the fields shrink as the fields themselves grow, the Lorentz forces grow even faster than the fields. Without special symmetries or other restrictions, the forces are amplified by a factor scaling as η^{-q}, where $1\leq q\leq 5/2$. This result implies that the problem of reconnection of a weak, disordered magnetic field embedded in a turbulent, highly conducting fluid cannot be solved self-consistently in the kinematic regime.

Although amplification of the field is generally undesirable in reconnection models for the reasons just described, it is exactly the effect sought in models of hydromagnetic dynamos. The dynamo properties of linear stagnation point flows have been investigated by Zel'dovich *et al.*;[14] see also Reference 15, by methods which overlap those used in this paper. These papers demonstrate the growth of magnetic energy with time induced by stagnation point flow, including randomly varying flow, but do not estimate the growth of Lorentz forces.

It is worth bearing in mind that these results are obtained only in the neighborhoods of stagnation points, not for globally bounded flows. This shortcoming is common to other reconnection models, which attempt to account for global effects through the choice of boundary conditions.[16]

It is often assumed that the effective resistivity of turbulent fluids is much larger than the Coulomb value, because action of the flow on the field produces small scale currents which are rapidly dissipated. This is exactly the process studied in this paper, although only for local flow models. While there can be no doubt that the field can be reconnected at a rate that depends only logarithmically on the resistivity, *in general* the fields are amplified and must eventually affect the flow. Thus, the results of this paper do not support the general concept of a large turbulent resistivity, but instead are consistent with arguments made by others based on numerical computation of global flows,[17–19] or on analytical calculations.[20–22] Reference 19, which discusses amplification of the field by a dynamo, shows that Lorentz forces reduce the stretching rate of a flow; because of the incompressibility condition, they must also reduce the shrinkage rate.

In the reconnection models presented here, the field is assumed to be initially weak, and the acceptability of the models is judged by whether it remains so. These models would be on shaky ground in systems such as stellar coronae and the bulk of the interstellar medium, which have strong fields and require dynamical reconnection theories. The models would be better applied to systems such as stellar interiors, accretion disks and the early universe, although each of these systems has distinctive features arising from other physical conditions.

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